In this project, we aim to characterize non cyber-insurable losses and the role these losses have on the user behavior to buy insurance on cyber-insurable losses. In general, there may be an ISP willing to insure a loss of a user that arises due to worm, virus, or botnet propagation, but not the same loss that arises due to system reliability or hardware/software related failures. As an example, there might be a hard-disk crash due to a security attack on a machine, whereas the same crash could also happen due to a bad hard-disk manufacture. Non-insurable losses could also be considered as special cases of insurable losses that are so small that the deductible on the insured product is greater than the loss and as a result there is no reimbursement by the insurance company. However, from a user perspective, it is quite susceptible to both types of losses. Assuming that we can distinguish between the two types of losses, we plan to address the following questions in our project. Our questions form the basis of the study on the user mindset to buying certain amounts of cyber-insurance under the realistic condition that users might incur uninsurable losses that result from non-security attacks.

- In the presence of non cyber-insurable losses and full overage at fair and unfair premiums, do risk averse users accept full cyber-insurance or are they more satisfied with co-insurance, where each user bears a certain liability for its own losses that occur due to security attacks.

- How does the demand amongst risk-averse users for cyber-insurance vary when risks due to non cyber-insurable losses increase. By the term ‘demand’, we imply the degree of cyber-insurance a user desires, i.e., full insurance or co-insurance. In regard to increase in risk due to non insurable losses, we consider three traditional settings most common to economic literature, 1) risk increase in a first order stochastic dominant sense, 2) risk increase in a second order stochastic dominant sense, and 3) risk increase in a Rothschild-Stiglitz sense.

- How does the demand amongst risk-averse users for cyber-insurance vary when insurable losses less than the deductible act as a special case of non-insurable losses, and the insurance is priced in both, a fair as well as in an unfair manner.

In addition to the above questions, we also plan to propose models to capture correlation amongst entities in the Internet. Correlation gives us a measure of risk interdependence when interdependence amongst Internet entities cannot be computed exactly.
1 Model

We have the following equation regarding the final wealth of an Internet user.

\[ W = W_0 + V - L_1 - L_2 + \theta(I(L_1) - P), \]

where \( W \) is random final wealth of a user, \( W_0 \) is its non-random initial wealth, \( V \) is the total value of the object subject to loss as a result of a security attack or a non-security attack. \( L_1 \) is a random variable denoting loss due to security attacks, \( L_2 \) is the random variable denoting loss due to non security attacks. \( I(L_1) \) is the cyber-insurance function that decides the amount of coverage to be provided in the event of a security-related loss, where \( 0 \leq I(L_1) \leq L_1 \). We assume that both \( L_1 \) and \( L_2 \) lie in the interval \([0, V]\). \( P \) is the premium charged to users in insurable losses and is defined as \( P = (1 + \lambda)E(I(L_1)) \). \( \lambda \) is the loading factor and is zero for fair premiums and greater than zero for unfair premiums. \( \theta \in [0, 1] \) is defined as the level of cyber-insurance opted for by a user. For example, a value of \( \theta = 0.6 \), implies that the user opts for insurance coverage of 60% of its losses and the rest 40% it considers as its own liability. This concept is termed as ‘co-insurance’ and is common in insurance policies. We assume that on the same object, there is either a loss due to security attacks or a loss due to non-security attacks. Both types of losses cannot inflict on an object simultaneously.

We define the expected utility of final wealth of an Internet user as

\[ E(W) = A + B + C + D, \] (1)

where

\[
A = \int \int_{0 < L_1 \leq V, L_2 = 0} u(W_0 + V - L_1 - L_2 + \theta(I(L_1) - P)) \cdot g(L_1, L_2) dL_1 \cdot dL_2,
\]

\[
B = \int \int_{0 < L_2 \leq V, L_1 = 0} u(W_0 + V - L_1 - L_2 + \theta(I(L_1) - P)) \cdot g(L_1, L_2) dL_1 \cdot dL_2,
\]

\[
C = \int \int_{0 < L_1, 0 < L_2} u(W_0 + V - L_1 - L_2 + \theta(I(L_1) - P)) \cdot g(L_1, L_2) dL_1 \cdot dL_2,
\]

and

\[
D = \beta \cdot u(W_0 + V - \theta \cdot P)
\]

We define the joint density probability density function of \( L_1 \) and \( L_2 \) as

\[
g(L_1, L_2) = \begin{cases} 
\alpha \cdot f_1(L_1) & 0 < L_1 \leq V, L_2 = 0 \\
(1 - \alpha - \beta) \cdot f_2(L_2) & 0 < L_2 \leq V, L_1 = 0 \\
0 & 0 < L_1 \leq V, 0 < L_2 \leq V
\end{cases}
\] (2)

where \( \alpha \)\(^1\) is the probability of loss occurring due to a security attack, and \( \beta \) is the probability of attack due to both, a security as well as a non security attack. \( u \) is a twice continuously differentiable risk-averse concave utility function of the user.

\(^1\)We plan to estimate \( \alpha \) using correlation models.
Based on the joint probability distribution function \( g() \), Equation 1 can be re-written as

\[
E(W) = A1 + B1 + C1,
\]

where

\[
A1 = \int_0^V u(W_0 + V - L_1 + \theta(I(L_1) - P))\alpha \cdot f_1(L_1)dL_1,
\]

\[
B1 = \int_0^V u(W_0 + V - L_2 - \theta(P))(1 - \alpha - \beta) \cdot f_2(L_2)dL_2,
\]

and

\[
C1 = \beta \cdot u(W_0 + V - \theta \cdot P)
\]

Now taking the first derivative of \( E(W) \) w.r.t. \( \theta \), and equating it to zero, we get the first order condition as

\[
\frac{dE(W)}{d\theta} = A2 + B2 + C2 = 0,
\]

where

\[
A2 = \int_0^V u'(W_0 + V - L_1 + \theta(I(L_1) - P))(I(L_1) - P)\alpha \cdot f_1(L_1)dL_1,
\]

\[
B2 = \int_0^V u'(W_0 + V - L_2 - \theta(P))(-P)(1 - \alpha - \beta) \cdot f_2(L_2)dL_2,
\]

and

\[
C2 = \beta \cdot u'(W_0 + V - \theta \cdot P)(-P)
\]

Now substituting \( I(L_1) = L_1 \) (indicating full coverage) and \( \theta = 1 \) (indicating no co-insurance) into the first order condition, we get

\[
\frac{dE(W)}{d\theta} = A3 + B3 + C3 = 0,
\]

where

\[
A3 = \int_0^V u'(W_0 + V - P)(L_1 - P)\alpha \cdot f_1(L_1)dL_1,
\]

\[
B3 = \int_0^V u'(W_0 + V - L_2 - P)(-P)(1 - \alpha - \beta) \cdot f_2(L_2)dL_2,
\]

and

\[
C3 = \beta \cdot u'(W_0 + V - P)(-P)
\]

Re-arranging the integrals we get

\[
A3 = u'(W_0 + V - P) \cdot \alpha \int_0^V (L_1 - P)f_1(L_1)dL_1,
\]

and

\[
B3 = (-P)(1 - \alpha - \beta)\int_0^V u'(W_0 + V - L_2 - P)f_2(L_2)dL_2.
\]
Now using the fact that $E(I(L_1)) = \alpha \cdot \int_0^V L_1 \cdot f_1(L_1) dL_1 = P$ (fair premiums), we have the following equation

$$
\frac{dE(W)}{d\theta} = A4 + B4,
$$

(6)

where

$$A4 = u'(W_0 + V - P)(1 - \alpha - \beta)P$$

and

$$B4 = (-P)(1 - \alpha - \beta) \int_0^V u'(W_0 + V - L_2 - \theta \cdot P) f_2(2L_2) dL_2,$$

Since a user has a risk-averse utility function, we have $u'(W_0 + V - L_2 - \theta \cdot P) > u'(W_0 + V - P) \forall L_2 > 0$. Thus, $\frac{dE(W)}{d\theta} < 0$ at $\theta = 1$. This indicates that the optimal value of $\theta$ is less than 1 for fair insurance premiums. On the other hand, even if we consider unfair premiums with a load factor $\lambda > 0$, we get $\frac{dE(W)}{d\theta} < 0$. Therefore in this case also the optimal value of theta is less than 1. Thus, we have the following theorem, the proof of which follows from analysis above.

**Theorem 1.** Internet users always choose less than full insurance, i.e., adopt co-insurance policies, when non-insurable losses exist, irrespective of whether the cyber-insurance premiums are fair or unfair.

**Theorem 2a.** When non-insurable losses are increased in a first order stochastic dominant sense, the demand for cyber-insurance amongst all risk-averse Internet users decreases.

**Proof.** Again consider the first order condition

$$
\frac{dE(W)}{d\theta} = A2 + B2 + C2 = 0,
$$

(7)

where

$$A2 = \int_0^V u'(W_0 + V - L_1 + \theta(I_L - P))(I(L) - P)\alpha \cdot f_1(L_1) dL_1,$$

$$B2 = \int_0^V u'(W_0 + V - L_2 - \theta(P))(-P)(1 - \alpha - \beta) \cdot f_2(2L_2) dL_2,$$

and

$$C2 = \beta \cdot u'(W_0 + V - \theta \cdot P)(-P).$$

We observe that when $L_2$ is increased in a first order stochastic dominant sense and $f_1(L_1$ and $\beta$ remain unchanged, the premium for insurance does not change. An increase in $L_2$ in the first order stochastic dominant sense increases the magnitude of $\int_0^V u'(W_0 + V - L_2 - \theta(P))(-P)(1 - \alpha - \beta) \cdot f_2(2L_2) dL_2$, whenever $u'(W_0 + V - L_2 - \theta(P))$ is increasing in $L_2$. This happens when $u(W)$ is concave, which is the exactly the case in our definition of $u$. Thus, an increase in $L_2$ in a first order stochastic dominant sense leads to the first order expression, $\frac{dE(W)}{d\theta}$, to become increasingly negative and results in reductions in $\theta$, implying the lowering
Theorem 2b. When non-insurable losses are increased in a Rothschild-Stiglitz sense, the demand for cyber-insurance amongst all prudent\footnote{A special category of risk averse users.} Internet users decreases.

Proof. We again consider the first order condition

$$\frac{dE(W)}{d\theta} = A2 + B2 + C2 = 0, \quad (8)$$

where

$$A2 = \int_0^V u'(W_0 + V - L_1 + \theta(I(L_1) - P))\alpha \cdot f_1(L_1)dL_1,$$

$$B2 = \int_0^V u'(W_0 + V - L_2 - \theta(P))(1 - \alpha - \beta) \cdot f_2(L_2)dL_2,$$

and

$$C2 = \beta \cdot u'(W_0 + V - \theta \cdot P)(-P)$$

When $L_2$ is increased in a Rothschild-Stiglitz sense, we need to observe the effect of the change of $f_2(L_2)$ on $\int_0^V u'(W_0 + V - L_2 - \theta(P))(1 - \alpha - \beta) \cdot f_2(L_2)dL_2$, which is determined by the concavity/convexity nature of $u'(W_0 + V - L_2 - \theta(P))$ w.r.t $L_2$. Since the Internet users are prudent, we have $u'''(W) > 0$ implying that $u'(W_0 + V - L_2 - \theta(P))$ is convex in $L_2$. Thus, the Rothschild-Stiglitz increases in $L_2$ results in the first order expression to be increasingly negative, thus lowering the demand for cyber-insurance amongst Internet users.

Theorem 3. When the risk due to non-insurable losses increases in either the first order stochastic dominant sense or the Rothschild-Stiglitz sense, the expected utility of final wealth for any cyber-insurance contract falls when compared to the alternative of no cyber-insurance for both, risk averse Internet users (in case of first order stochastic dominance sense) as well as prudent Internet users (in case of risk increase in the Rothschild-Stiglitz sense).

Proof. The expected utility of any cyber-insurance contract is given by the following

$$E(W) = A1 + B1 + C1, \quad (9)$$

where

$$A1 = \int_0^V u(W_0 + V - L_1 + \theta(I(L_1) - P))\alpha \cdot f_1(L_1)dL_1,$$

$$B1 = \int_0^V u(W_0 + V - L_2 - \theta(P))(1 - \alpha - \beta) \cdot f_2(L_2)dL_2,$$
and

\[ C1 = \beta \cdot u(W_0 + V - \theta \cdot P) \]

When \( \theta = 0 \) (the case for no cyber-insurance), \( E(W) \) reduces to

\[ E(W) = A1' + B1' + C1', \quad (10) \]

where

\[ A1' = \int_0^V u(W_0 + V - L_1)\alpha \cdot f_1(L_1)\,dL_1, \]

\[ B1' = \int_0^V u(W_0 + V - L_2)(1 - \alpha - \beta) \cdot f_2(L_2)\,dL_2, \]

and

\[ C1' = \beta \cdot u(W_0 + V) \]

Increases in \( L_2 \) affect only the second terms in each of these utility expressions. Thus, we need to consider the change in the second order terms in the two utility expressions to observe the impact of the increase in \( L_2 \). The difference in the second order terms is given as

\[ \int_0^V [u(W_0 + V - L_2 - \theta(P)) - u(W_0 + V - L_2)](1 - \alpha - \beta) \cdot f_2(L_2)\,dL_2, \]

which evaluates to

\[ \int_0^V [u(W_0 + V - L_2 - \theta(P)) - u(W_0 + V - L_2)](1 - \alpha - \beta) \cdot f_2(L_2)\,dL_2, \]

where \([u(W_0 + V - L_2 - \theta(P)) - u(W_0 + V - L_2)]\) is decreasing in \( L_2 \) under risk aversion and concave under user prudence. Thus, increases in \( L_2 \) in the first order stochastic dominant sense or in the Rothschild-Stiglitz sense reduces the expected utility of cyber-insurance relative to no cyber-insurance.