Anisotropic Normal Estimation for Sparse Point Clouds

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ABSTRACT

This technical report presents a normal estimation method for point clouds with low sampling density and sharp features. To achieve the best trade-off between quality and performance, normal of a point on smooth regions is estimated using an isotropic neighborhood with constant size; but normal of a point near features are evaluated with an anisotropic neighborhood from which the local tangent plane can be reliably approximated.

1. INTRODUCTION

Normal information of point clouds is important to a number of point-based applications, such as refinement, rendering, and surface reconstruction. Hoppe et al. pioneered regression based normal estimation. They apply PCA over a constant k nearest neighbors of a point and take as normal the eigenvector corresponding to the smallest eigenvalue of the covariance matrix. The methods proposed in [5, 6] are weighted variances of Hoppe’s approach. Guennebaud et al. provide stable results even for low sampling rates by fitting an algebraic sphere to the local neighborhood of a point, and taking the sphere’s gradient as the normal. To provide the best trade-off between the ability to handle noise and to recover small details, the work of [8] uses the optimal neighborhood size adaptive to the local noise scale, curvature and sampling density. These approaches can produce good results on smooth regions, however, they fail to provide reliable normals near edges as normals are estimated by isotropic neighbors from both sides.

Several other normal estimation approaches are based on anisotropic neighborhood. Hu et al. compute normals by applying a bilateral estimation scheme on the adaptively constructed multi-layer neighbors. Yoon et al. apply the ensemble technique from statistics to improve the robustness of. Recently, work of [11] uses a RANSAC-like method to detect the best local tangent plane for normal estimation. The approach of [12] estimates the final normal as the maximum of the discrete probability distribution of possible normals based on a robust Randomized Hough Transform. These algorithms are capable to deal with points located in singular regions in presence of noise and outliers. However, they are either computationally expensive or require dense sampling or both.

Li et al. observe that, for each point, in most cases an anistropic neighborhood can be determined from which the local tangent plane can be reliably approximated. To achieve the best trade-off between quality and performance for low density point data, we design a normal estimation algorithm that uses a constant isotropic neighborhood for points on smooth regions and an anisotropic neighborhood for points near features.

2. NORMAL ESTIMATION

The point cloud taken as input to our method is a set of 3D points \( P = \{ p_1, p_2, ..., p_N \} \), \( p_i \in \mathbb{R}^3 \) without any connectivity information. \( N_k(p) \) is used to denote k-nearest neighbors of a point \( p \in P \), which collects k neighbors nearest to \( p \) regardless of the distance.

**Smoothness Test** Given a point \( p \) and its k-nearest neighbors \( N_k(p) \), the surface variance at \( p \) as introduced in [13] is:

\[
\nu(p) = \frac{\lambda_0}{\lambda_0 + \lambda_1 + \lambda_2},
\]

where \( \lambda_i \) are the eigenvalues of the covariance matrix evaluated over \( p \) and \( N_k(p) \) with \( \lambda_0 \leq \lambda_1 \leq \lambda_2 \). If \( \nu(p) \leq \nu_{\text{smooth}} \), we consider \( p \) lies on a smooth area and take \( N_k(p) \) as its neighborhood for normal estimation. The threshold \( \nu_{\text{smooth}} \in [0, 1/3] \) is user adjustable.
Anisotropic Neighborhood Selection  For a point \( p \) failing the smoothness test, we adapt the idea of Gauss map clustering\(^{14} \) to find its best anisotropic neighborhood where all neighbors share the similar tangent plane with \( p \). This is motivated by the fact that these neighbor points will cluster together on the Gauss map.

The point \( p \) together with any two neighbors \( p_i \) and \( p_j, i < j, i, j \in [0, k) \) form a triangle \( \triangle_{ij} \). The normal vector of \( \triangle_{ij} \) can be estimated as

\[
n_{ij} = \overrightarrow{pp_i} \times \overrightarrow{pp_j}
\]

As \( n_{ij} \) only determines the direction not the orientation, we choose from the pair \( \{n_{ij}, -n_{ij}\} \) the one closer to the positive \( z \)-axis as the normal of \( \triangle_{ij} \). The set of normals of all possible \( C_{k,2} \) triangles is denoted as \( N \). Same as\(^{12} \) it is unnecessary to check collinearity as the random normal generated by near collinear points does not affect much the final clustering result.

The algorithm of\(^{4} \) maps \( N \) to the unit sphere centered at \( p \) via a projection operation. We skip this projection and analyze directly the original \( N \) which naturally maps to the unit sphere centered at the origin, since the clustering behavior is irrelevant to the sphere’s location. Note that only the half sphere with positive \( z \) is covered by \( N \).

Similar to\(^{14} \) the hierarchical agglomerative clustering method of\(^{15} \) is applied to group normals of \( N \) based on geodesic distance on the unit Gauss sphere. The clustering algorithm stops when the distance between two clusters exceeds a threshold \( \eta \in [0, \frac{\pi}{2}] \). We now analyze \( \zeta \) clusters containing more than \( c_{\min} \) members:

- If \( \zeta = 0 \) or \( \zeta > 4 \), \( p \) is considered as a noise, we take \( N_4(p) \) as its neighborhood. The number of four is from the assumption that most data sets generally don’t have more than four sharp features meeting in one point\(^{14} \)

- If \( 1 \leq \zeta \leq 4 \), \( p \) is considered to be near or on a sharp feature. All points of a cluster (a cluster member \( n_{ij} \) is associated with two neighbor points \( p_i \) and \( p_j \)) form an anisotropic neighborhood candidate for \( p \). We can fit a least square plane to \( p \) and points of a cluster. Based on the distance from \( p \) to the fitted plane, the clusters closest to \( p \) are identified.

  - If only one such cluster, neighbor points of this cluster form the best anisotropic neighborhood of \( p \), as depicted in Figure 1(a).

  - If two or more clusters, \( p \) can be categorized to any of these clusters, i.e., \( p \) lies exactly on a sharp feature. In this case (Figure 1(b)), the normal of \( p \) is ambiguous. To avoid abrupt normal changes along a feature edge, we don’t choose any one of the clusters but use \( N_4(p) \) instead.

For the value of \( k \), 8 and 32 are considered as the lower and upper bounds to deliver acceptable results\(^{12} \). Our experiment uses 16 for \( k \). \( \eta \) is the sensitivity parameter for Gauss map clustering, the smaller the value, the more points including noise will be detected as near sharp features. A relatively low value (0.1 - 0.4) works for our data. The value of \( c_{\min} \) is locally adjusted as half of the members of the largest cluster of \( p \), which is more effective than a global setting.

![Figure 1](image-url)  
Figure 1. Anisotropic neighborhood selection for point \( p \) (blue) sits (a) near the edge, and (b) on the edge, using 16-nearest neighbors. Red for selected neighbors and black for unselected neighbors.

Normal Estimation  Finally, PCA is applied over point \( p \) and its neighbors (either isotropic or anisotropic) selected as above, and the normal at \( p \) takes the eigenvector corresponds to the least eigenvalue of the covariance matrix as in\(^{4} \)
3. EXPERIMENT

To test the effectiveness of the presented normal estimation approach, we compare the normal difference between the estimation and the ground truth on a set of synthetic point clouds, which are sampled from wedges with various angles: 30°, 90°, 150°. The original point clouds are uniformly sampled with spacing 0.4 in a 10 × 10 bounding box. The noisy point clouds are generated by perturbing each point a random vector with size 20% and 50% of the point spacing. We visualize the data with Root Mean Square with threshold ($RMS_{\tau}$) errors as in:

$$RMS_{\tau} = \sqrt{\frac{1}{|P|} \sum_{p \in P} v_p^2}$$

where \(v_p = n_{p,\text{ref}} n_{p,\text{est}}\) if \(n_{p,\text{ref}} n_{p,\text{est}} < \tau\), and \(v_p = \frac{\pi}{2}\) otherwise. \(n_{p,\text{ref}}\) is the reference ground truth normal at \(p\) and \(n_{p,\text{est}}\) is the estimated normal at \(p\).

Figure 2 compares $RMS_{10}$ of normals estimated over isotropic and our anisotropic neighborhood with $k = 24$. It shows that normals of points near edges are obviously better with anisotropic neighborhood estimation for either acute or obtuse angles, under moderate noise.

![Figure 2. Normal estimation comparison of isotropic and anisotropic neighborhoods, under different noise levels. Wedges have various angles: 30° (left two columns), 90° (middle two columns) and 150° (right two columns). Points with errors larger than 10° are colored in red, colors of the rest points are interpolated between green (good) and blue (bad). The values of sensitivity parameter $\eta$ for clustering are given for normal estimation with anisotropic neighborhood.](image)

REFERENCES


